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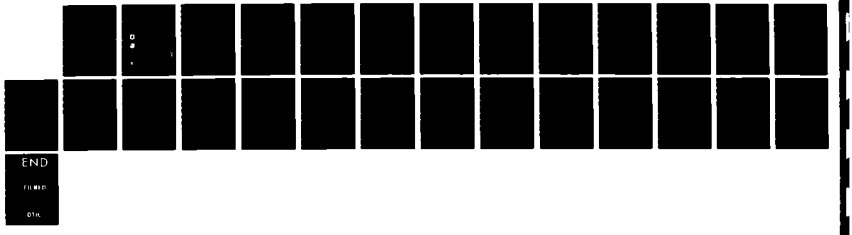

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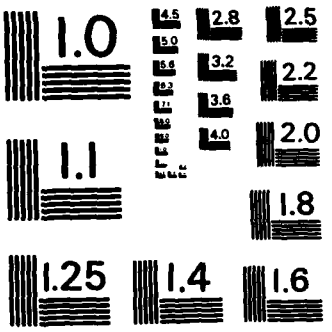

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A Parameterized Procedure for Determining Real Height From Ionograms by Use of Generalized Parabolic Profiles

MILTON M. KLEIN

8 May 1985

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A direct method for obtaining real heights from virtual height data was obtained by developing a series of generalized parabolic profiles with which a given ionogram may be compared. The present method simplifies a previous method in which parameterized polynomials were utilized and provides a more realistic fit to ionospheric electron density profiles. Application of the present method to ionograms from Boulder, Colorado, and Garchy, France, gave results that were in good agreement with previously reported accurate numerical calculations.

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A Parameterized Procedure for Determining Real Height From Ionograms by Use of Generalized Parabolic Profiles

1. INTRODUCTION

In previous reports[1,2] an analytic procedure was developed for determining real heights from ionograms by directly solving the integral relation between real height and virtual height. The real height could then be obtained by a direct integration of the virtual height over the range of frequencies encountered. In a later report[3] the method was applied to typical ionograms; the required integration being performed numerically. In addition, a simplified procedure was developed in which an ionogram could be fitted to a parametric series of polynomials and the integration required for determination of real height performed analytically. In the present report we generalize this procedure by modeling electron density profiles in terms of a parameterized series of generalized parabolic profiles. The profiles are then converted to virtual height or ionogram form. The real height corresponding to a given ionogram is obtained by comparison of the ionogram

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1. Klein, M.M. (1979) A Method for Direct Determination of Real Height From Virtual Height Data for the Auroral Region of the Ionosphere, AFGL-TR-79-0276, AD A083139.

2. Klein, M.M. (1981) A Simplified Procedure for Direct Determination of Real Height From Virtual Height Data, AFGL-TR-81-0070, AD A104374.

3. Klein, M.M. (1983) Application of a Direct Method for Real Height Determination to Two Ionograms, AFGL-TR-83-0064, AD A130430.

with the parameterized profiles. Since the real height for a given parameterized profile is related in a very simple way to its virtual height, the integration process required for the previous polynominal method is hereby avoided. In addition, since the parabolic profile has a vertical slope at the maximum electron density, it provides a better fit to ionospheric data than a polynomial. Additional keywords: ionosonde data; computations; equations.

2. DETERMINATION OF VIRTUAL HEIGHT FOR PARABOLIC PROFILES

The parabolic profile frequently utilized for ionospheric analysis is of the form

$$\left(\frac{h_m - h}{h_m - h_o}\right)^2 = \left(1 - \frac{N}{N_p}\right) , \tag{1}$$

where N is the electron density, N_p the maximum value of N, h_m the maximum height of the profile, and h_o the initial height at which N goes to zero. This profile is characterized by a small but non-zero slope at h_o and a vertical slope at N_p. It is convenient to write Eq. (1) in terms of frequencies by

$$\left(\frac{h_m - h}{h_m - h_o}\right)^2 = \left(1 - \frac{f_N^2}{f_p^2}\right) = (1 - b^2 X) , \tag{2}$$

where f_N is the plasma frequency, f_p its maximum value, X is the ratio f_N^2/f^2, b the ratio f/f_p, and f is the probing frequency. The foregoing profile is characterized by a very small rate of increase of height (and hence of virtual height) at low frequencies. We shall refer to this variation of f_N as a f_N^2 profile. In order to cover ionograms characterized by a more rapid increase of height at low frequencies, it is useful to also consider the profile characterized by a linear variation of frequency (referred to as a linear f_N or f_N profile).

$$\left(\frac{h_m - h}{h_m - h_o}\right)^2 = \left(1 - \frac{f_N}{f_p}\right) = 1 - bX^{1/2} . \tag{3}$$

We now generalize the profiles given by Eqs. (2) and (3) by replacing the power 2 by a parameter n. As n increases the slope of the profile decreases. We have found that a range of n from 1 to 4 is generally sufficient for the ionograms encountered in practice. Accordingly, Eqs. (2) and (3) are replaced by

$$(h_m - h)^n = a(1 - b^2X) \tag{4}$$

$$(h_m - h)^n = a(1 - bX^{1/2}) \quad , \tag{5}$$

where $a = (h_m - h_o)$. A plot of Eq. (4) exhibiting the form of the generalized parabolic curves for several values of n is given in Figure 1. For subsequent use, we write Eqs. (4) and (5) in the form

$$h_v - h_o = a[1 - (1 - b^2)^{1/n}] \tag{6}$$

$$h_v - h_o = a[1 - (1 - b)^{1/n}] \quad , \tag{7}$$

where h_v is the height corresponding to a maximum probing frequence f_v.

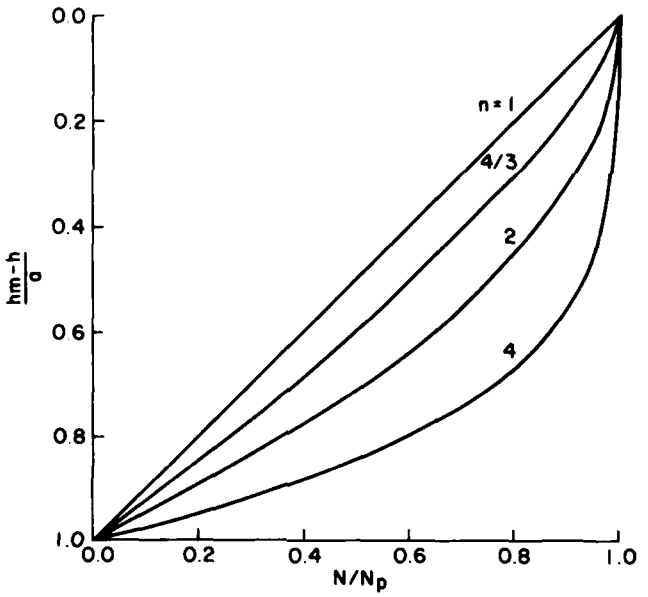

Figure 1. Plot of Generalized Parabolic Curves for Several Values of Parameter n. The electron density N has been normalized to its maximum value N_p

The virtual height h'(f) corresponding to a density distribution X is given by

$$h'(f) = \frac{d}{df}(fI) = \frac{d}{db}(bI) \quad , \tag{8}$$

where

$$I = \int_0^{h_r} (1 - X)^{\alpha/2}\, dh \quad , \tag{9}$$

h_r is the reflection height, and α is a parameter incorporating the effect of the magnetic field. A plot of α as a function of frequency for several values of the angle α between the ray and the magnetic field is given in Reference 3, Figure 3. For the profiles defined by Eqs. (4) and (5), we may write Eq. (9) in the form

$$f_N^2 \cdot I = \frac{ab^2}{n} \int_0^1 \frac{(1 - X)^{\alpha/2}}{(1 - b^2X)^{1-1/n}}\, dX \tag{10}$$

$$f_N \cdot J = \frac{ab}{n} \int_0^1 \frac{(1 - u^2)^{\alpha/2}}{(1 - bu)^{1-1/n}}\, du \quad , \tag{11}$$

where $u = f_N/f$, and we now distinguish between the f_N^2 and f_N cases by use of I and J. In general, the integrals in Eqs. (10) and (11) are not analytically tractable for arbitrary α, but may be approximated by integrals that can be evaluated. We first write Eqs. (10) and (11) in the form

$$f_N^2 \cdot I = \frac{ab^2}{n} \int_0^1 \frac{(1 - X)^{\frac{\alpha}{2} - 1 + \frac{1}{n}} (1 - X)^{1 - \frac{1}{n}}}{(1 - b^2X)^{1 - \frac{1}{n}}}\, dX \tag{12}$$

$$f_N \cdot J = \frac{ab^2}{n} \int_0^1 \frac{(1 - u)^{\frac{\alpha}{2} - 1 + \frac{1}{n}} (1 + u)^{\frac{\alpha}{2}} (1 - u)^{1 - \frac{1}{n}}}{(1 - bu)^{1 - \frac{1}{n}}}\, du \tag{13}$$

and note that the quantities $(1 - X)^{(\alpha/2)-1+n}$, $(1 - u)^{(\alpha/2)-1+n}$, and $(1 + u)^{\alpha/2}$ are slowly varying functions over the range of integration and may be replaced by average values. We accordingly approximate Eqs. (12) and (13) by

$$f_N^2 \cdot \quad I = \frac{ab^2}{n} g \int_o^1 \frac{(1 - X)^{1 - \frac{1}{n}}}{(1 - b^2X)^{1 - \frac{1}{n}}} \, dX \tag{14}$$

$$f_N \cdot \quad J = \frac{ab}{n} g_1 g_2 \int_o^1 \frac{(1 - u)^{1 - \frac{1}{n}}}{(1 - bu)^{1 - \frac{1}{n}}} \, du \quad , \tag{15}$$

where

$$g = \int_o^1 (1 - X)^{\frac{\alpha}{2} - 1 + \frac{1}{n}} \, dX \tag{16}$$

$$g_1 = \int_o^1 (1 - u)^{\frac{\alpha}{2} - 1 + \frac{1}{n}} \, du = \frac{1}{\frac{\alpha}{2} + \frac{1}{n}} = g \tag{17}$$

$$g_2 = \int_o^1 (1 + u)^{\frac{\alpha}{2}} \, du = \frac{2^{\frac{\alpha}{2} + 1} - 1}{\frac{\alpha}{2} + 1} \quad . \tag{18}$$

A numerical check of the exact and approximate integrals for several values of b shows that the approximate forms are within a few percent of the corresponding exact integrals.

Carrying through the differentiation indicated in Eq. (8), the virtual height h'(f) takes the form,

$$f_N^2 \cdot \quad h'(f) = \frac{ab^2}{n} g \left[\left(1 + \frac{2}{n} \right) I_1 + 2 \left(1 - \frac{1}{n} \right) I_2 \right] \quad , \tag{19}$$

where

$$I_1 = \int_o^1 \frac{(1 - X)^{1 - \frac{1}{n}}}{(1 - b^2X)^{1 - \frac{1}{n}}} \, dX \tag{20}$$

$$I_2 = \int_o^1 \frac{(1 - X)^{1 - \frac{1}{n}}}{(1 - b^2X)^{2 - \frac{1}{n}}} \, dX \tag{21}$$

$$f_N \, , \quad h'(f) = \frac{ab}{n} \, g_1 g_2 \left[\left(1 + \frac{1}{n} \right) J_1 + \left(1 - \frac{1}{n} \right) J_2 \right] \, , \tag{22}$$

where

$$J_1 = \int_o^1 \frac{(1 - u)^{1 - \frac{1}{n}}}{(1 - bu)^{1 - \frac{1}{n}}} \, du \tag{23}$$

$$J_2 = \int_o^1 \frac{(1 - u)^{1 - \frac{1}{n}}}{(1 - bu)^{2 - \frac{1}{n}}} \, du \, . \tag{24}$$

Since we shall be taking rational values of n, the power $1 - 1/n$ may be written as p/q, where p and q are integers. Rationalizing the integrands in I and J by use of

$$y^q = \left(\frac{1 - X}{1 - b^2X} \right)^p \tag{25}$$

$$y^q = \left(\frac{1 - u}{1 - b^2u} \right)^p \tag{26}$$

transforms the I and J integrals to the forms,

$$f_N^2 \, , \quad I_1 = (1 - b^2) \, q \int_o^1 \frac{y^{p+q-1}}{(1 - b^2 y^q)^2} \, dy \tag{27}$$

$$I_2 = q \int_o^1 \left(\frac{y^{p+q+1}}{1 - b^2 y^q} \right) dy \tag{28}$$

$$f_N \cdot \quad J_1 = (1 - b)\, q \int_0^1 \frac{y^{p+q-1}}{(1 - by^q)^2}\, dy \tag{29}$$

$$J_2 = q \int_0^1 \frac{y^{p+q-1}}{(1 - by^q)}\, dy \quad , \tag{30}$$

which can easily be integrated.

3. EVALUATION OF VIRTUAL HEIGHT FOR SQUARE f_N PROFILE

3.1 $n = 4$

The I integrals and h'(f) are given by

$$I_1 = 4(1 - b^2) \int_0^1 \frac{y^6}{(1 - b^2 y^4)}\, dy \tag{31}$$

$$I_2 = 4 \int_0^1 \frac{y^6}{(1 - b^2 y^4)}\, dy \tag{32}$$

$$h'(f) = \frac{3}{8}\, a\, b^2\, g(I_1 + I_2) \quad , \tag{33}$$

which upon evaluating I_1 and I_2 yields,

$$h'(f) = \frac{3}{8}\, a\, g \left\{ -\frac{1}{3} + \left[\frac{1 + 3b^2}{4\, b^{3/2}} \ln \left(\frac{1 + b^{1/2}}{1 - b^{1/2}} \right) - 2 \tan^{-1} b^{1/2} \right] \right\} \quad . \tag{34}$$

For small b, Eq. (34) has the series form

$$h'(f) = \frac{3}{8}\, a\, g \left(\frac{8}{7}\, b^2 + \frac{40}{77}\, b^4 + \frac{56}{165}\, b^6 + \frac{24}{95}\, b^8 + \ldots \right) \quad . \tag{35}$$

3.2 n = 2

For this case we obtain

$$I_1 = 2(1 - b^2) \int_o^1 \frac{y^2}{(1 - b^2 y^2)^2} \, dy \tag{36}$$

$$I_2 = 2 \int_o^1 \frac{y^2}{1 - b^2 y^2} \, dy \tag{37}$$

$$h'(f) = \frac{1}{2} a b^2 g(2I_1 + I_2) \tag{38}$$

$$h'(f) = \frac{1}{2} a b g \ln \left(\frac{1 + b}{1 - b} \right) \tag{39}$$

$$h'(f) = a g \left(b^2 + \frac{b^4}{3} + \frac{b^6}{5} + \frac{b^8}{7} + \ldots \right) \tag{40}$$

3.3 n = 4/3

$$I_1 = 4(1 - b^2) \int_o^1 \frac{y^4}{(1 - b^2 y^4)^2} \, dy \tag{41}$$

$$I_2 = 4 \int_o^1 \frac{y^4}{(1 - b^2 y^4)} \, dy \tag{42}$$

$$h'(f) = \frac{3}{8} a b^2 g(5I_1 + I_2) \tag{43}$$

$$h'(f) = \frac{3}{8} a g \left\{ 1 + \frac{1}{4} (5 - b^2) b^{3/2} \left[\ln \frac{1 + b^{1/2}}{1 - b^{1/2}} + 2 \tan^{-1} b^{1/2} \right] \right\} \tag{44}$$

$$h'(f) = \frac{3}{8} a g \left(\frac{24}{5} b^2 + \frac{8}{9} b^4 + \frac{56}{117} b^6 + \frac{72}{221} b^8 + \ldots \right) \tag{45}$$

3.4 n = 1

For this case, the integral I in Eq. (10) can be evaluated directly to yield,

$$I = \frac{a b^2}{1 + \alpha/2} \quad , \tag{46}$$

which gives a virtual height

$$h'(f) = \frac{3ab^2}{1 + \alpha/2} \quad . \tag{47}$$

4. EVALUATION OF VIRTUAL HEIGHT FOR LINEAR f_N PROFILE

4.1 n = 4

The J integrals and h'(f) are given by (note that the J integrals are replicas of the I integrals with b^2 replaced by b)

$$J_1 = 4(1 - b) \int_0^1 \frac{y^6}{(1 - b y^4)^2} \, dy \tag{48}$$

$$J_2 = 4 \int_0^1 \frac{y^6}{(1 - b y^4)} \, dy \tag{49}$$

$$h'(f) = \frac{1}{16} a b g_1 g_2 (5J_1 + 3J_2) \tag{50}$$

$$h'(f) = \frac{1}{16} a g_1 g_2 \left\{ 1 + \frac{3}{4 b^{3/4}} (5b - 1) \left[\ln \frac{1 + b^{1/4}}{1 - b^{1/4}} - 2\tan^{-1} b^{1/4} \right] \right\} \tag{51}$$

$$h'(f) = \frac{1}{16} a g_1 g_2 \left(\frac{32}{7} b + \frac{144}{77} b^2 + \frac{64}{55} b^3 + \frac{16}{19} b^4 + \ldots \right) \tag{52}$$

4.2 n = 2

$$J_1 = 2(1 - b) \int_0^1 \frac{y^2}{(1 - b y^2)^2} \, dy \tag{53}$$

$$J_2 = 2 \int_0^1 \frac{y^2}{(1 - by^2)} \, dy \tag{54}$$

$$h'(f) = \frac{1}{4} a b g_1 g_2 (3J_1 + J_2) \tag{55}$$

$$h'(f) = \frac{1}{4} a g_1 g_2 \left[1 + \frac{1}{2b^{1/2}} (3b - 1) \ln \frac{1 + b^{1/2}}{1 - b^{1/2}} \right] \tag{56}$$

$$h'(f) = \frac{1}{4} a g_1 g_2 \left(\frac{8}{3} b + \frac{4}{5} b^2 + \frac{16}{35} b^3 + \frac{20}{63} b^4 + \ldots \right) \tag{57}$$

4.3 n = 4/3

$$J_1 = 4(1 - b) \int_0^1 \frac{y^4}{(1 - by^4)^2} \, dy \tag{58}$$

$$J_2 = 4 \int_0^1 \frac{y^4}{(1 - by^4)} \, dy \tag{59}$$

$$h'(f) = \frac{3}{16} a b g_1 g_2 (7J_1 + J_2) \tag{60}$$

$$h'(f) = \frac{3}{16} a g_1 g_2 \left\{ 3 + \frac{7b - 3}{4b^{1/4}} \left[\ln \frac{1 + b^{1/4}}{1 - b^{1/4}} + 2\tan^{-1} b^{1/2} \right] \right\} \tag{61}$$

$$h'(f) = \frac{3}{16} a g_1 g_2 \left(\frac{32}{5} b + \frac{16}{5} b^2 + \frac{64}{117} b^3 + \frac{80}{221} b^4 + \ldots \right) \tag{62}$$

4.4 n = 1

The integral J in Eq. (9) can here be evaluated exactly to yield

$$J = \frac{\pi^{1/2}}{1+\alpha} \frac{\Gamma\left(1+\frac{\alpha}{2}\right)}{\Gamma\left(\frac{1+\alpha}{2}\right)} \tag{63}$$

and a virtual height

$$h'(f) = 2ab \frac{\pi^{1/2}}{1+\alpha} \frac{\Gamma\left(1+\frac{\alpha}{2}\right)}{\Gamma\left(\frac{1+\alpha}{2}\right)} . \tag{64}$$

5. APPROXIMATE SOLUTION FOR n CLOSE TO UNITY

The integrations required in the foregoing analysis may be considerably simplified when n is near one. For the f_N^2 profile we write Eqs. (20) and (21) in the form

$$I_1 = \int_0^1 \left[1 - \frac{(1-b^2)X}{1-b^2X}\right]^{\epsilon} dX \tag{65}$$

$$I_2 = \int_0^1 \left[1 - \frac{(1-b^2)X}{1-b^2X}\right]^{\epsilon} \frac{dX}{(1-b^2X)} . \tag{66}$$

where $\epsilon = 1 - \frac{1}{n}$. Since $\frac{(1-b^2)X}{1-b^2X} \leq 1$ we expand the binominal term in the powers of ϵ and upon integration obtain, to first order in ϵ,

$$b^2 I_1 = b^2 + \frac{\epsilon}{b^2}(1-b^2)[b^2 + \ln(1-b^2)] \tag{67}$$

$$b^2 I_2 = -\ln(1-b^2) - \frac{\epsilon}{b^2}[(1-b^2)\ln(1-b^2) + b^2] . \tag{68}$$

For small b, I_1 and I_2 may be written as

$$b^2 I_1 = b^2 \left[1 - \frac{\epsilon}{2} \left(1 - \frac{1}{3} b^2 - \frac{1}{6} b^4 - \frac{1}{10} b^6 + \ldots \right) \right] \tag{69}$$

$$b^2 I_2 \quad b^2 + \frac{b^4}{2} + \frac{b^6}{3} + \frac{b^8}{4} + \frac{b^{10}}{5} + \ldots - \frac{\epsilon}{2} \left(b^2 + \frac{b^4}{3} + \frac{b^6}{6} + \frac{b^8}{10} + \ldots \right) . \tag{70}$$

As indicated previously, the J_1 and J_2 integrals are duplicates of the I_1 and I_2 integrals with b^2 replaced by b. We then have, for the linear f_N profile,

$$bJ_1 \quad b + \frac{\epsilon}{b} (1 - b)[b + \ln (1 - b)] \tag{71}$$

$$bJ_2 = b - \frac{\epsilon}{b} [(1 - b) \ln (1 - b) + b] \tag{72}$$

$$bJ_1 \quad b - \frac{\epsilon b}{2} \left(1 - \frac{1}{3} b - \frac{1}{6} b^2 - \frac{1}{10} b^3 + \ldots \right) \tag{73}$$

$$bJ_2 = b + \frac{b^2}{2} + \frac{b^3}{3} + \frac{b^4}{4} + \frac{b^5}{5} - \frac{\epsilon}{2} \left(b + \frac{b^2}{3} + \frac{b^3}{6} + \frac{b^4}{10} + \ldots \right) . \tag{74}$$

A numerical check of the approximate solutions developed in this section were within a few percent of their more exact counterparts in Section 4, even for ϵ as high as 0.75. The good accuracy obtained, even for moderate values of ϵ, is due to the small value of the integrated term. It is estimated that the first neglected term in the expansion is, in general, only a few percent of the first (ϵ free) term.

6. DETERMINATION OF REAL HEIGHT

The calculated values of h'(f) are presented as a function of frequency for the f_N^2 profiles in Figures 2 through 4 and for the linear f_N profiles in Figures 5 through 7. The frequency f has been normalized to $x = f/f_v$, where f_v is the maximum value of the probing frequency f for a given curve. The parameter $b_v = f_v/f_p$ has been used to characterize the set of curves for a given figure. The virtual height has been normalized to its maximum value $h'(f_v)$. Values of h'(f), aside from the coefficients a and g, may be obtained directly from Figures 8 and 9,

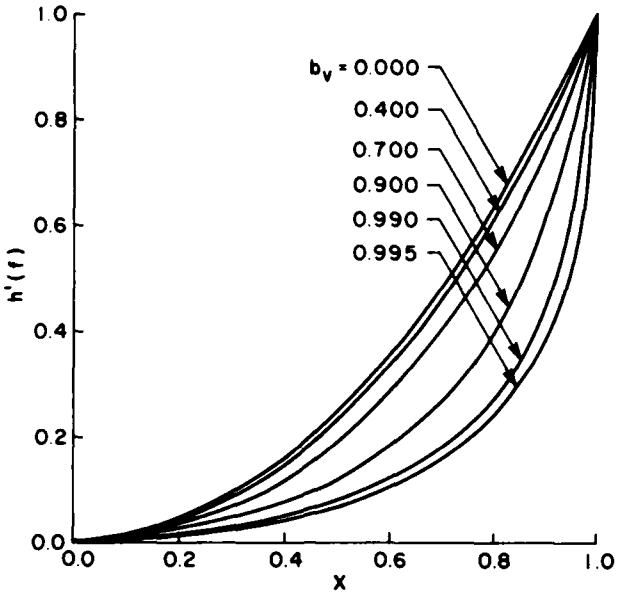

Figure 2. Virtual Height h'(f) for the f_N^2 Profiles as a Function of Frequency x for Several Values of Maximum Frequency Parameter b; Parameter n = 4. The virtual height h'(f) has been normalized to its maximum value h'(f_v)

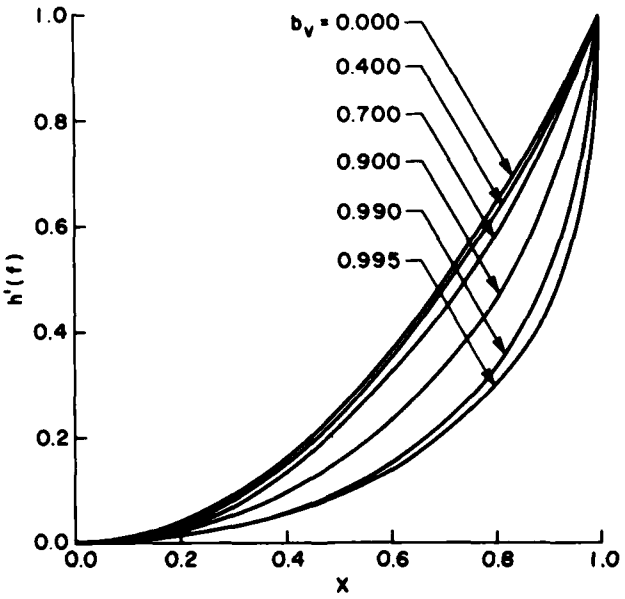

Figure 3. Virtual Height h'(f) for the f_N^2 Profiles as a Function of Frequency x for Several Values of Maximum Frequency Parameter b; Parameter n = 2. The virtual height h'(f) has been normalized to its maximum value h'(f_v)

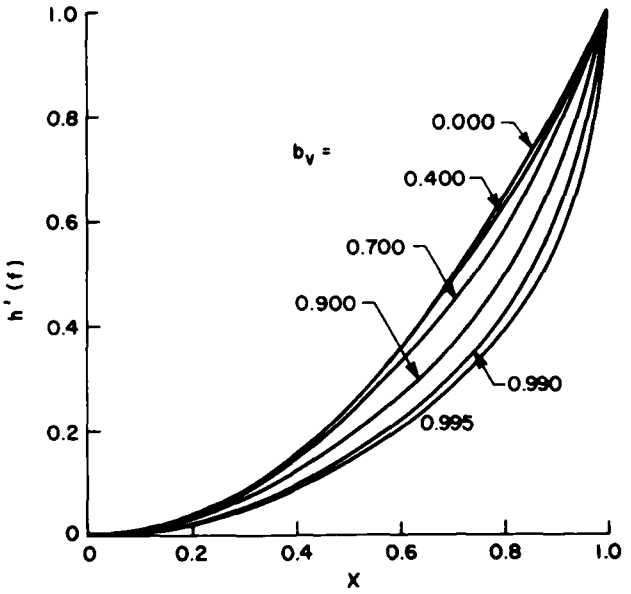

Figure 4. Virtual Height h'(f) for the f_N^2 Profiles as a Function of Frequency x for Several Values of Maximum Frequency Parameter b; Parameter n = 4/3. The virtual height h'(f) has been normalized to its maximum value h'(f_v)

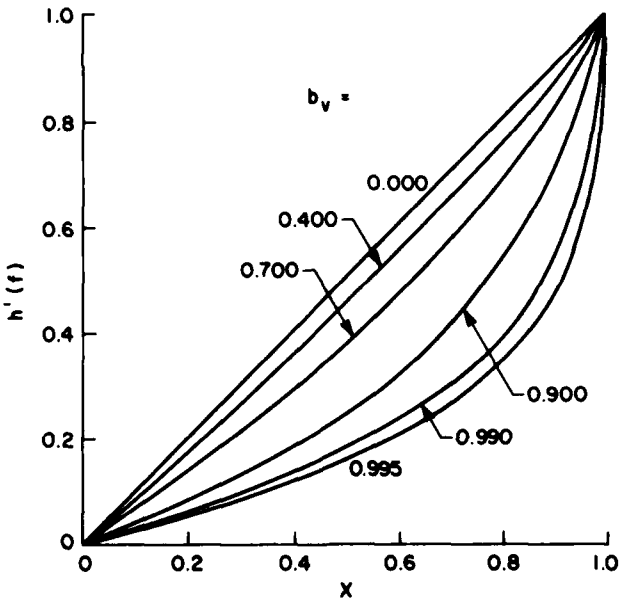

Figure 5. Virtual Height h'(f) for Linear f_N Profiles as a Function of Frequency x for Several Values of Maximum Frequency Parameter b; Parameter n = 4. The virtual height h'(f) has been normalized to its maximum value h'(f_v)

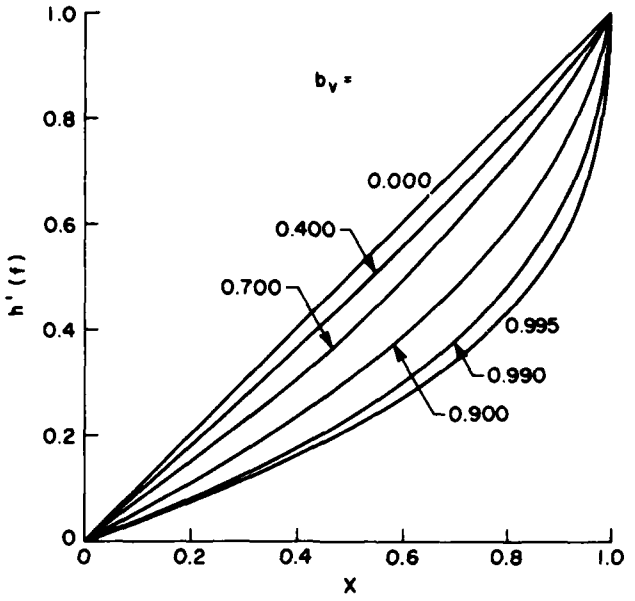

Figure 6. Virtual Height h'(f) for Linear f_N Profiles as a Function of Frequency x for Several Values of Maximum Frequency Parameter b; Parameter n = 2. The virtual height h'(f) has been normalized to its maximum value h'(f_v)

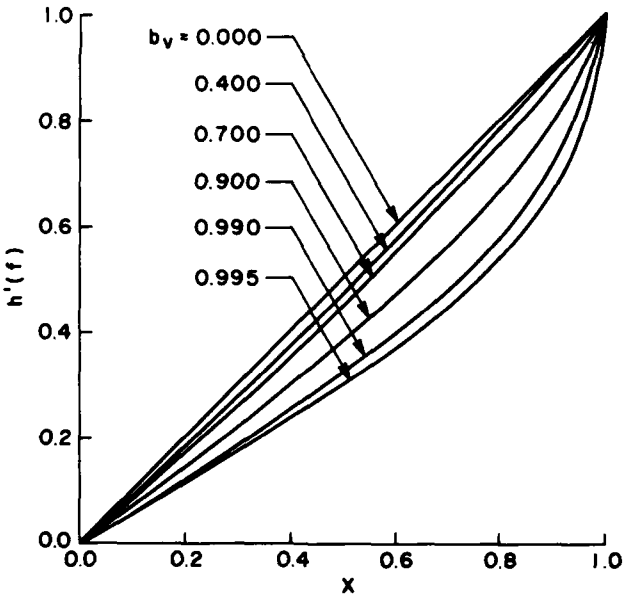

Figure 7. Virtual Height h'(f) for Linear f_N Profiles as a Function of Frequency x for Several Values of Maximum Frequency Parameter b; Parameter n = 4/3. The virtual height h'(f) has been normalized to its maximum value h'(f_v)

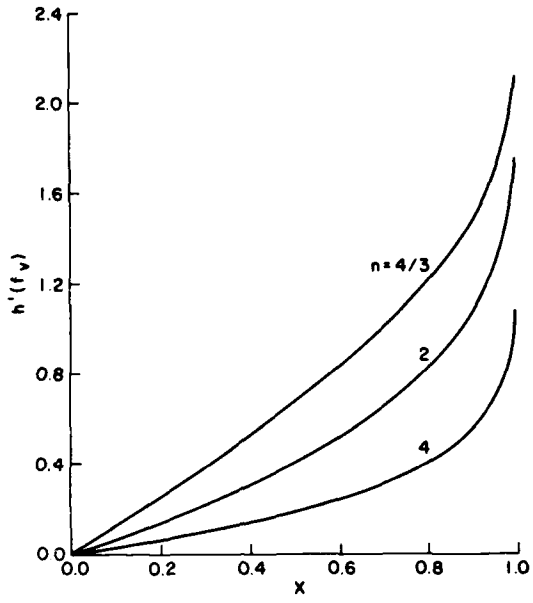

Figure 8. Factor for Obtaining h'(f) From Normalized Values of h'(f) for f_N^2 Profiles in Figures 2 through 4

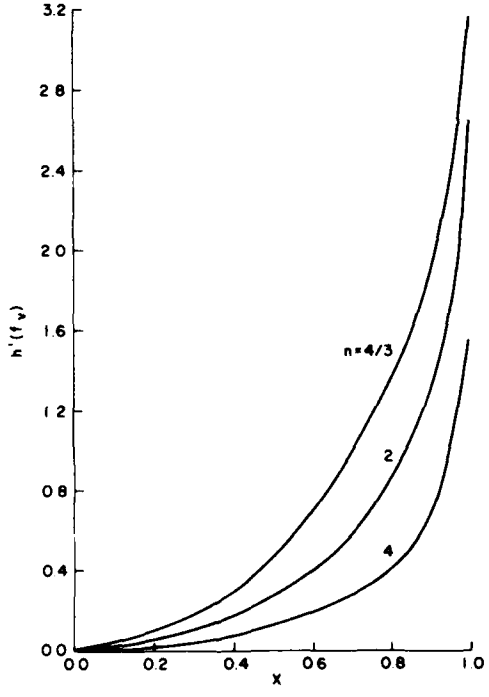

Figure 9. Factor for Obtaining h'(f) From Normalized Values of h'(f) for Linear f_N Profiles in Figures 5 Through 7

which give, for specific values of n and b_v, the factor $h'(f_v)$, by which the normalized values of h'(f) are multiplied. The a and g coefficients may be incorporated later for a specific probing frequency.

The ionograms chosen for determination of real height are the Boulder and Garchy ionograms previously utilized in Reference 3. In the present report we omit the determination of initial height since this method has been presented in detail in Reference 3. The parameters required for the calculation of real height are obtained from a comparison of a given ionogram with the h'(f) curves developed herein. The h'(f) curves selected for the Boulder ionograms are plotted in Figures 10 and 11, and those for the Garchy ionograms in Figures 12 and 13. For greater clarity the ionogram data are plotted as discrete points.

As an example, the method of determining the real height h is presented for the Boulder ionogram at the higher frequency $f = 7.65$ MHz. (For convenience, values of h and h' are here taken relative to h_o):

From Reference 3 $h' = 180$ km
$h = 76$ km

From Figure 10 $n = 4$, linear f_N profile, $b_v = 0.90$

From Figure 9 $h'(f_v) = 0.557$.

We now determine the parameter a for the parabolic profile by comparing the virtual heights for the ionogram and the parabolic profile:

From Reference 3, Figure 3 $\alpha = 0.7$.

From Eqs. (17) and (18) $g_1 = 1.667$, $g_2 = 1.148$

$$h' = g_1 g_2 a h'(f_v)$$

$$180 = (1.667)(1.148)(0.557)a$$

$$a = 168.8 \text{ km} .$$

For the parabolic profile the real height is, from Eq. (7),

$$h = a[1 - (1 - b_v)^{1/4}]$$

$$h = 168.8(0.438)$$

$$= 73.9 \text{ km} .$$

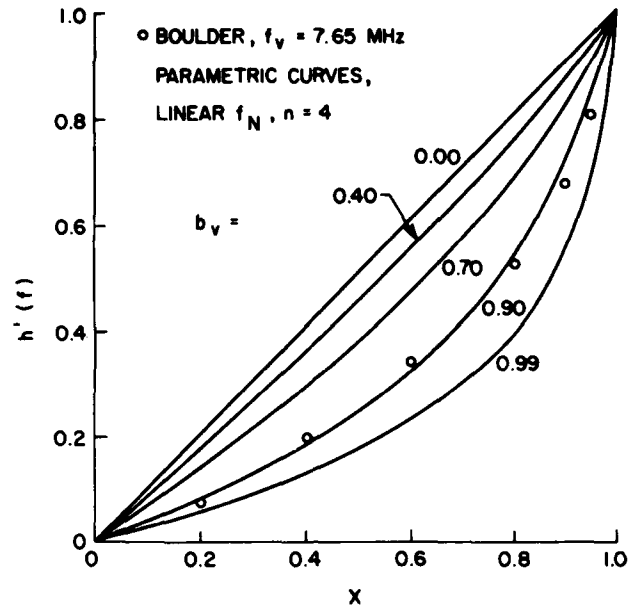

Figure 10. Comparison of Boulder Ionogram for f_v = 7.65 MHz With h'(f) Curves; Linear f_N Profiles, n = 4

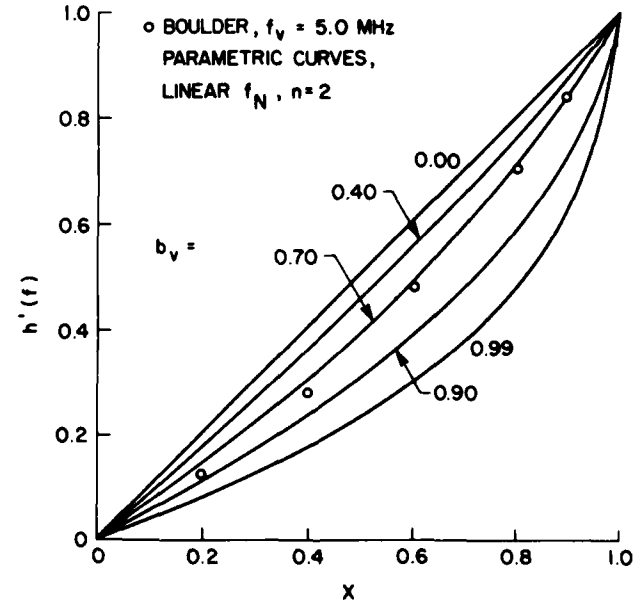

Figure 11. Comparison of Boulder Ionogram for f_v = 5.0 MHz With h'(f) Curves; Linear f_N Profiles, n = 2

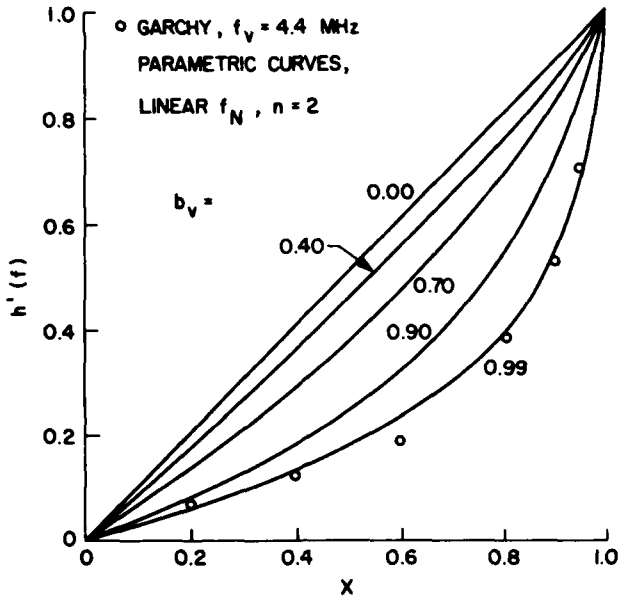

Figure 12. Comparison of Garchy Ionogram for f_v = 4.4 MHz With h'(f) Curves; Linear f_N Profiles, n = 4

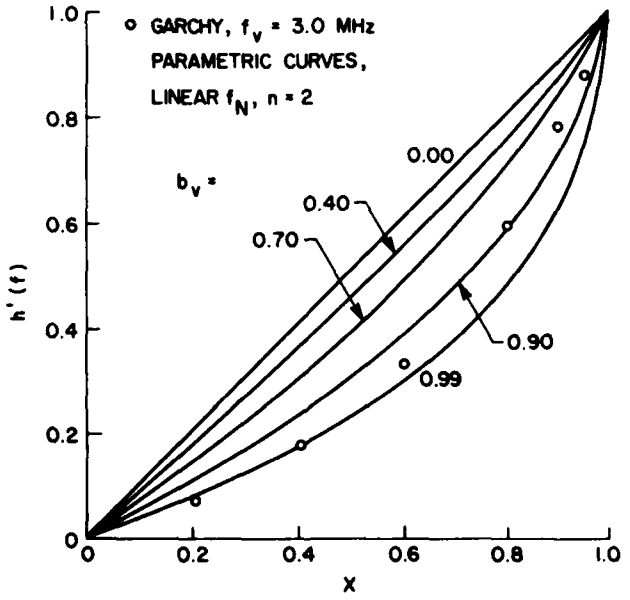

Figure 13. Comparison of Garchy Ionogram for f_v = 3.0 MHz With h'(f) Curves; Linear f_N Profiles, n = 2

In a similar manner, the following results were obtained for the Boulder and Garchy ionograms:

Boulder, f_v = 7.65 MHz, α = 0.7, b_v = 0.9, n = 4 (linear f_N)
h(calc.) = 73.9 km, h(Ref. 3) = 76 km

Boulder, f_v = 5.0 MHz, α = 0.6, b_v = 0.75, n = 2 (linear f_N)
h(calc.) = 34.1 km, h(Ref. 3) = 33 km

Garchy f_v = 4.4 MHz, α = 0.6, b_v = 0.99, n = 4 (linear f_N)
h(calc.) = 92.0 km, h(Ref.3) = 98 km

Garchy, f_v = 3.0 MHz, α = 0.5, b_v = 0.95, n = 2 (linear f_N)
h(calc.) = 27.9 km, h(Ref.3) = 31 km

As seen from Figures 10 through 13, the ionograms could be fitted reasonably well to the parabolic profiles, except for the Garchy ionogram at the lower frequency of 3.0 MHz. For this case, the ionogram bends too rapidly at the lower values of h' before following the trend of the parabolic profiles. However, other trial fits of the ionograms to the parabolic profiles, yielded real heights fairly close to the values obtained herein, indicating that a reasonable fit should yield a representative value of the true height. Improvements in the fitting of the ionograms to the parabolic profiles and in the accuracy of the real heights can presumably be achieved by calculating for several intermediate powers of f_N and for a greater range of n.

7. CONCLUDING REMARKS

A direct method for obtaining real heights from virtual height data has been obtained by developing a series of generalized parabolic profiles with which a given ionogram may be compared. Ionograms from Boulder, Colorado, and Garchy, France, were chosen for the calculations. The results obtained were in fairly good agreement with those obtained from previously reported accurate numerical calculations.

The present method affords a generalization of the procedure presented in Reference 3, in which virtual height data was represented by a parameterized series of polynomials from which real heights could be obtained by the evaluation of simple integrals. Since the parabolic profile has a vertical slope at its maximum electron density, it affords a more realistic fit to ionospheric electron density data

than a polynomial. The method utilizing the generalized parabolic profiles yields the real height from the virtual height data or ionogram by a simple direct calculation.

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